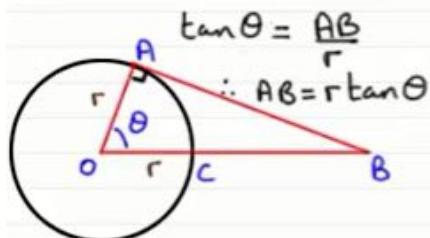




Small angle approximations for $\sin(x)$, $\cos(x)$ and $\tan(x)$



$$\tan \theta = \frac{AB}{r}$$

$$\therefore AB = r \tan \theta$$

Area $\Delta OAB >$ Area of sector $OAC >$ Area ΔOAC

$$\frac{1}{2}r^2 \tan \theta > \frac{1}{2}r^2 \theta > \frac{1}{2}r^2 \sin \theta$$

$$\therefore \tan \theta > \theta > \sin \theta \quad \textcircled{1}$$

From \textcircled{1} $\div \sin \theta$

$$\therefore \frac{1}{\cos \theta} > \frac{\theta}{\sin \theta} > 1$$

as $\theta \rightarrow 0$, $\cos \theta \rightarrow 1$

$$\therefore \lim_{\theta \rightarrow 0} \frac{1}{\cos \theta} \rightarrow 1$$

$$\therefore \frac{\theta}{\sin \theta} \rightarrow 1$$

$$\therefore \sin \theta \approx \theta$$

From \textcircled{1} $\div \tan \theta$

$$\therefore 1 > \frac{\theta}{\tan \theta} > \cos \theta$$

as $\theta \rightarrow 0$, $\cos \theta \rightarrow 1$

$$\therefore \lim_{\theta \rightarrow 0} \frac{\theta}{\tan \theta} \rightarrow 1$$

$$\therefore \tan \theta \approx \theta$$

since $\cos \theta = \sqrt{1 - \sin^2 \theta}$

$$= (1 - \theta^2)^{1/2}$$

$$= 1 - \frac{1}{2}\theta^2 - \frac{\theta^4}{8} - \dots$$

$$\approx 1 - \frac{\theta^2}{2} \quad \text{for small } \theta$$

for small θ , measured in radians

$$\sin \theta \approx \theta$$

$$\tan \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

With the acknowledgement of [Exam Solutions](#).

Find lots more revision sheets on [Air Maths Tuition](#).

[This Video](#)

