Air maths tuition
Interact. engage and perform

Rcos(x-alpha) method
| Past Paper Question | C3 Edexcel Jan 2013 Q4

(a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
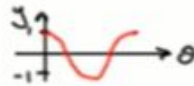
Give the value of α to 3 decimal places.

(b) $\quad p(\theta) = \dfrac{4}{12 + 6\cos\theta + 8\sin\theta}$, $\qquad 0 \leqslant \theta \leqslant 2\pi$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

$$A\sin\theta \pm B\cos\theta \equiv R\sin(\theta \pm \alpha)$$
$$A\cos\theta \pm B\sin\theta \equiv R\cos(\theta \mp \alpha)$$
$$\text{where } R = \sqrt{A^2 + B^2}, \quad \alpha = \tan^{-1}\frac{B}{A}$$

$6\cos\theta + 8\sin\theta \equiv A\cos\theta + B\sin\theta$

where $A = 6$, $B = 8$

$\therefore R = \sqrt{6^2 + 8^2}$ $\qquad \alpha = \tan^{-1}\frac{8}{6}$

$\quad = 10$ $\qquad\qquad\qquad = 0.92729...$

$\qquad\qquad\qquad\qquad\qquad = 0.927 \text{ (3dp)}$

$\therefore 6\cos\theta + 8\sin\theta \equiv 10\cos(\theta - 0.927)$

i) $p(\theta) = \dfrac{4}{12 + 10\cos(\theta - 0.927)}$

$\text{max } p(\theta)$: when $\cos(\theta - 0.927) = -1$

$\therefore \text{max } p(\theta) = \dfrac{4}{12 - 10} = 2$

ii) when $\theta - 0.92729... = \pi$

$\therefore \theta = 4.0688...$

$\therefore \theta = 4.07 \text{ (3dp)}$

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