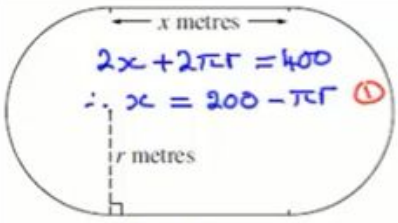
Air maths tuition

Interact, engage and perform

Optimisation | Past Paper Question | P1 CIE Nov 2013 Q8(ii)



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

(i) Show that the area, A m^2, of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]

(ii) Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

$2x + 2\pi r = 400$

$\therefore x = 200 - \pi r$ ①

$A = 400r - \pi r^2$

$\therefore \frac{dA}{dr} = 400 - 2\pi r$

At a stationary point $\frac{dA}{dr} = 0$

$\therefore 400 - 2\pi r = 0 \Rightarrow r = \frac{200}{\pi}$

Sub into ①

$x = 200 - \pi\left(\frac{200}{\pi}\right)$

$= 0$

Nature stationary point

$\frac{d^2A}{dr^2} = -2\pi < 0$

$\therefore$ a maximum

r	60	$\frac{200}{\pi}$	70
$\frac{dA}{dr}$	23.0...	0	-39.8...

Slope

$\therefore$ a maximum

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