



Air maths tuition

Interact, engage and perform

Harmonic form $A\sin(x)+B\cos(x)$ | Past Paper Question | C3 OCR June 2012 Q8(i)(ii)(a)

(i) Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

$$3\sin\theta + 4\cos\theta \equiv A\sin\theta + B\cos\theta$$

where $A = 3$ and $B = 4$

$$\therefore R = \sqrt{3^2 + 4^2} \quad \alpha = \tan^{-1} \frac{4}{3}$$

$$= 5 \quad \quad \quad = 53.1301\dots$$

$$A\sin\theta \pm B\cos\theta \equiv R\sin(\theta \pm \alpha)$$

$$A\cos\theta \pm B\sin\theta \equiv R\cos(\theta \mp \alpha)$$

where $R = \sqrt{A^2 + B^2}$, $\alpha = \tan^{-1} \frac{B}{A}$

$$\therefore 3\sin\theta + 4\cos\theta \equiv 5\sin(\theta + 53.1^\circ) \text{ (1dp)}$$

(ii) Hence (a) solve the equation $3\sin\theta + 4\cos\theta + 1 = 0$, giving all solutions for which $-180^\circ < \theta < 180^\circ$. [4]

$$3\sin\theta + 4\cos\theta + 1 = 0$$

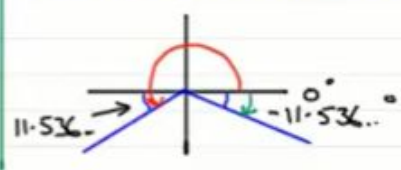
$$\therefore 5\sin(\theta + 53.1301\dots^\circ) = -1$$

$$\therefore \sin(\theta + 53.1301\dots^\circ) = -0.2$$

$$\therefore \theta + 53.1301\dots^\circ = \sin^{-1}(-0.2)$$

$$= -11.536\dots^\circ, 191.536\dots^\circ$$

New range:
 $-126.86\dots < \theta + 53.1301\dots < 233.13\dots$



$$\therefore \theta = -64.666\dots, 138.405\dots$$

$$\therefore \theta = -64.7^\circ, 138.4^\circ \text{ (1dp)}$$

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